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# A New Odderon Solution in Perturbative QCD

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**Abstract**

We present and discuss a new bound state solution of the three gluon system in perturbative QCD. It carries the quantum numbers of the odderon, has intercept at one and couples to the impact factor $\gamma^* \to \eta_c$.

1. The unitarization of the BFKL Pomeron [1] presents one of the major tasks in QCD. After the successful calculation of the NLO corrections to the BFKL kernel [2] and recent progress in analyzing its properties [3], there are several directions in going beyond the two-gluon ladder approximation. One of them investigates multigluon compound states. After the first formulation of the BKP equations [4] it was found that, in the large-$N_c$ limit, their solutions have remarkable mathematical properties [5], and the hamiltonian is the same as for the integrable spin chain [6]. The existence of integrals of motion [5] and the duality symmetry [7] provide powerful tools in analysing the spectrum of energy eigenvalues. Another line of research investigates the transition between states with different numbers of gluons [8, 9, 10, 11].

As the first step beyond the two-gluon system (i.e. the BFKL equation) the spectrum of the three gluon system (odderon [12]) has attracted much attention recently. Apart from the theoretical interest in understanding the dynamics of the n-gluon states with $n > 2$, there is the long-standing odderon problem which provides interest from the phenomenological side. After several variational studies an eigenfunction of the integral of motion [5] with the odderon intercept slightly below one was constructed by Janik and Wosiek [13] (see also [7]) and verified by Braun et al [14]. From the phenomenological side, a possible signature of the odderon in deep inelastic scattering at HERA has been investigated by several authors [15, 16]. In this context also the coupling of the odderon to the $\gamma^* \to \eta_c$ vertex has been calculated [15]. Another piece of information relevant for the three gluon channel is the Pomeron → two-odderon vertex which has been obtained from an analysis of the six-gluon

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state [11]. Its momentum structure coincides with the momentum dependence found in [15].

In this letter we present a new explicit solution to the three gluon system which carries the quantum numbers of the odderon and has interecept one. It is derived from the momentum structure found in [11] and [15]. This solution can be interpreted as the reggeization of a d-reggeon in QCD (the even-signature color octet reggeon which is degenerate with the odd-signature reggeized gluon), which interacts with the reggeized gluon. Our new solution can also be obtained by applying a duality transformation to the antisymmetric solution found in [17]. From the phenomenological point of view, this new solution seems to be more important than the previous one: its intercept is higher than that of the totally symmetric odderon solution of [13]. In the final part of this letter we shall explicitely show how our solution couples to the $\gamma^* \to \eta_c$-vertex.

2. Let us begin with the coupling of three gluons to external particles. In order to be able to apply perturbative QCD we should start from a virtual photon, $\gamma^*$, which splits into two quarks. For the elastic impact factor $\gamma^* \to \gamma^*$ it was shown in [9] that in the t-channel with three gluons the bootstrap property of the gluon reduces the number of reggeized gluons to two, i.e. there is no state with three reggeized gluons. Therefore, a nonzero coupling of a three gluon t-channel to external particles needs an outgoing state whose parity is even, i.e. opposite to the photon. The easiest candidate is the $\gamma^* \to \eta_c$-vertex which has been calculated in [15] (Fig1a). Its momentum structure (as a function of transverse momenta) has the following the form:

$$\Phi^{(i)}_{\gamma\to\eta_c} \sim g_s^3 \epsilon_{ij} \frac{q_j}{\boldsymbol{q}^2} \left( \sum_{(123)} \frac{(\boldsymbol{k}_1 + \boldsymbol{k}_2 - \boldsymbol{k}_3)\cdot \boldsymbol{q}}{Q^2 + 4m_c^2 + (\boldsymbol{k}_1 + \boldsymbol{k}_2 - \boldsymbol{k}_3)^2} - \frac{\boldsymbol{q}^2}{Q^2 + 4m_c^2 + \boldsymbol{q}^2} \right) . \tag{1}$$

Here $\boldsymbol{q} = \boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3$, the sum extends over the cyclic permutations of (1,2,3), and we have left out an overall factor in front which is not important for our present discussion. The color structure is simply given by the symmetric structure constants $d^{a_1 a_2 a_3}$. By introducing the short hand notation

$$\varphi_-^{(i)}(\boldsymbol{k}, \boldsymbol{k}') = g_s^2 \epsilon_{ij} \frac{q_j}{\boldsymbol{q}^2} \frac{(\boldsymbol{k} - \boldsymbol{k}')(\boldsymbol{k} + \boldsymbol{k}')}{Q^2 + 4m_c^2 + (\boldsymbol{k} - \boldsymbol{k}')^2} \, , \tag{2}$$

we rewrite (1) as

$$\Phi^{(i)}_{\gamma\to\eta_c} \sim g_s \Big( \sum_{(123)} \varphi_-^{(i)}(\boldsymbol{k}_1 + \boldsymbol{k}_2, \boldsymbol{k}_3) - \varphi_-^{(i)}(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3, \boldsymbol{0}) \Big). \tag{3}$$

The function $\varphi_-$ is antisymmetric under the exchange of its two arguments. It is easy to see that $\Phi$ vanishes as one of the transverse momenta $\boldsymbol{k}_i$ goes to zero (with fixed $\boldsymbol{q}$). The full sum of the cyclic permutations is symmetric under the exchange of any pair of momenta $(\boldsymbol{k}_i, \boldsymbol{k}_j)$, but because of the antisymmetry of $\varphi_-$ its symmetry structure is more involved and will be discussed below.

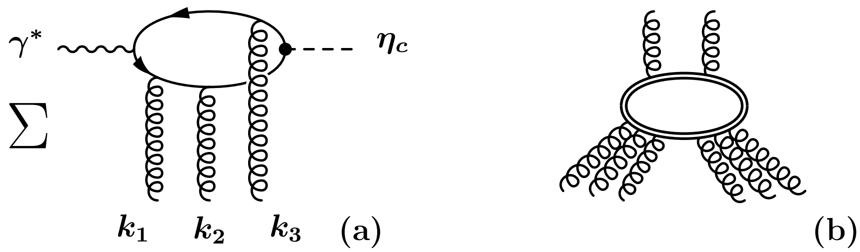


Figure 1: (a) The $\Phi^{(i)}_{\gamma\to\eta_c}$ impact factor corresponds to the sum of the graphs with different gluon configurations. (b) The effective Pomeron $\to$ two-Odderon vertex $W$. Each group of three outcoming gluons is in a totally symmetric color singlet state.

Interesting enough, the same momentum structure (3) has also been found in the new Pomeron $\to$ two-odderon vertex of [11] (Fig.1b). Starting from eq.(6.12) in [11], one first expresses [19] the function $W(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3;\boldsymbol{k}_4,\boldsymbol{k}_5,\boldsymbol{k}_6)$ in terms of the function $G(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)$ which was first introduced in [9] and further investigated in [20]. As an important property we note that this $G$-function vanishes if either $\boldsymbol{k}_1$ or $\boldsymbol{k}_3$ goes to zero. Next we introduce the function

$$\begin{aligned}\varphi_{--}(\boldsymbol{k}_1,\boldsymbol{k}_2;\boldsymbol{k}_3,\boldsymbol{k}_4) &= \\ g_s^4\Big(G(\boldsymbol{k}_1,\boldsymbol{k}_2+\boldsymbol{k}_3,\boldsymbol{k}_4) - G(\boldsymbol{k}_2,\boldsymbol{k}_1+\boldsymbol{k}_3,\boldsymbol{k}_4) &- G(\boldsymbol{k}_1,\boldsymbol{k}_2+\boldsymbol{k}_4,\boldsymbol{k}_3) + G(\boldsymbol{k}_2,\boldsymbol{k}_1+\boldsymbol{k}_4,\boldsymbol{k}_3)\Big). \end{aligned} \tag{4}$$

Then the Pomeron $\to$ two-odderon vertex $W$ takes the following form:

$$\begin{aligned} &W(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3;\boldsymbol{k}_4,\boldsymbol{k}_5,\boldsymbol{k}_6) \sim \\ &\sum_{(123)}\sum_{(456)}\varphi_{--}(\boldsymbol{k}_1+\boldsymbol{k}_2,\boldsymbol{k}_3;\boldsymbol{k}_4,\boldsymbol{k}_5+\boldsymbol{k}_6) - \sum_{(123)}\varphi_{--}(\boldsymbol{k}_1+\boldsymbol{k}_2,\boldsymbol{k}_3;\boldsymbol{0},\boldsymbol{k}_4+\boldsymbol{k}_5+\boldsymbol{k}_6) \\ &-\sum_{(456)}\varphi_{--}(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3,\boldsymbol{0};\boldsymbol{k}_4,\boldsymbol{k}_5+\boldsymbol{k}_6) + \varphi_{--}(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3,\boldsymbol{0};\boldsymbol{0},\boldsymbol{k}_4+\boldsymbol{k}_5+\boldsymbol{k}_6) \end{aligned} \tag{5}$$

Now one easily sees that the momentum structure of the three gluon systems (123) or (456) is the same as in (3). Again we have the property that $W$ vanishes as any of the $\boldsymbol{k}_i$ goes to zero (from (5) one sees immediately that $W\to 0$ when $\boldsymbol{k}_i\to 0$, with the total odderon momenta $\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3$ and $\boldsymbol{k}_4+\boldsymbol{k}_5+\boldsymbol{k}_6$ being kept fixed). The color structure is given by the product of the two d-tensors: $d^{a_1a_2a_3}d^{a_4a_5a_6}$.

3. Starting from this momentum structure it is easy to find a solution for the three gluon system. For simplicity we return to the impact factor $\Phi_{\gamma\to\eta_c}$ in (3). Disregarding, for the moment, the last term which serves as a subtraction constant, we consider the convolution of $\sum_{(123)}\varphi_{-}(\boldsymbol{k}_1+\boldsymbol{k}_2,\boldsymbol{k}_3)$ with the kernel for the three gluon state:

$$K_{(123)} = \sum_{(ij)} K_{(ij)} \tag{6}$$

where

$$K_{(12)} = K(\boldsymbol{k}_1, \boldsymbol{k}_2; \boldsymbol{k}'_1, \boldsymbol{k}'_2) \tag{7}$$

is the LO BFKL kernel which includes the gluon trajectory functions. We find

$$\left( K_{(123)} \otimes \frac{1}{\boldsymbol{k}_1^2 \boldsymbol{k}_2^2 \boldsymbol{k}_3^2} \sum_{(123)} \varphi_-(\boldsymbol{k}_1 + \boldsymbol{k}_2, \boldsymbol{k}_3) \right) (\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) =$$
$$\sum_{(123)} \left( K_{(12)} \otimes \frac{1}{\boldsymbol{k}_1^2 \boldsymbol{k}_2^2} \varphi_-(\boldsymbol{k}_1, \boldsymbol{k}_2) \right) (\boldsymbol{k}_1 + \boldsymbol{k}_2, \boldsymbol{k}_3) \tag{8}$$

In deriving this result it is important to use the color structure $d^{a_1 a_2 a_3}$, the antisymmetry of $\varphi_-$, and the bootstrap property of the BFKL kernel. The latter is a relation which guarantees that production amplitudes with the gluon quantum number in their $t$ channels used for the construction of the absortive part are characterized by just a single reggeized gluon exchange (at leading and next-to-leading orders). The convolution symbol $\otimes$ denotes the integral over transverse momenta, and we have explicitly written the gluon propagators between $\varphi_-$ and the BFKL kernel. For the moment we ignore the fact that the integral in (8) is infrared singular, since the function $\varphi_-$ does not vanish as one of its argument goes to zero. Next we replace the function $\varphi_-(\boldsymbol{k}, \boldsymbol{q} - \boldsymbol{k})$ by the BFKL (normalized) eigenfunction $E^{(\nu,n)}(\boldsymbol{k}, \boldsymbol{q} - \boldsymbol{k})$; for odd values of the conformal spin $n$ this function is odd under the interchange of its arguments $\boldsymbol{k}$ and $\boldsymbol{q} - \boldsymbol{k}$. This leads to the following definition:

$$E_3^{(\nu,n)}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) = g_s \frac{N_c}{\sqrt{N_c^2 - 4}} \frac{1}{\sqrt{-3\chi(\nu, n)}} \sum_{(123)} \frac{(\boldsymbol{k}_1 + \boldsymbol{k}_2)^2}{\boldsymbol{k}_1^2 \boldsymbol{k}_2^2} E^{(\nu,n)}(\boldsymbol{k}_1 + \boldsymbol{k}_2, \boldsymbol{k}_3), \tag{9}$$

where

$$\chi(\nu, n) = \frac{N_c \alpha_s}{\pi} \left( 2\psi(1) - \psi(\frac{1 + |n|}{2} + i\nu) - \psi(\frac{1 + |n|}{2} - i\nu) \right) \tag{10}$$

is the characteristic function of the BFKL kernel, and the global color structure is again given by $d^{a_1 a_2 a_3}$. The function $E_3^{(\nu,n)}$ satisfies (8), but since $E^{(\nu,n)}$ is an eigenfunction of the BFKL kernel, we can go one step further and obtain

$$K_{(123)} \otimes E_3^{(\nu,n)} = \chi(\nu, n) E_3^{(\nu,n)}. \tag{11}$$

The leading eigenvalue for $n = \pm 1$, $\nu = 0$ lies at zero, i.e. in the angular momentum plane the rightmost singularity lies at $j = 1$. Hence this solution dominates over the totally symmetric solution of [13]. Let us remark that in (9) we have included a normalization factor such that the norm of $E_3^{(\nu,n)}$ turns out to be equal to the norm of $E^{(\nu,n)}$.

Property (8) can be viewed as the reggeization of the d-reggeon: starting from the initial condition (as given by (3)) and evolving the three gluon state with the help of the kernel (6), the identity (8) tells us that the three gluon system "collapses" into a two-reggeon

state, where one reggeon is the well-known reggeized gluon (in the antisymmetric color octet reperesentation), the other one a d-reggeon (belonging to the symmetric color octet representation). The full state is in a color singlet, but it has odd C-parity. This situation can be compared with the three gluon state in the Pomeron channel (even C-parity) discussed in [8, 9]: here the initial condition is given by the $D_{(3,0)}$ function. The three gluons also evolve and "collapse" into two reggeized gluons. The main difference lies in the evolution which, in the Pomeron channel, contains also a transition kernel $2 \to 3$ gluons. Such a kernel is absent in the odderon channel.

4. For our further discussion it is convenient to switch to configuration space. We will show that the new solution (9) can be also be obtained from another solution which has been found recently in [17]. Using the Moebius invariance of the Hamiltonian for the compound states $\Psi_{m,\widetilde{m}}(\boldsymbol{\rho}_0)$ of the three reggeized gluons in LLA in the impact parameter space $\boldsymbol{\rho}$, we can write the ansatz for the corresponding wave function

$$f_{m,\widetilde{m}}(\boldsymbol{\rho}_1,\, \boldsymbol{\rho}_2,\, \boldsymbol{\rho}_3;\, \boldsymbol{\rho}_0) = \left(\frac{\rho_{23}}{\rho_{20}\rho_{30}}\right)^m \left(\frac{\rho^*_{23}}{\rho^*_{20}\rho^*_{30}}\right)^{\widetilde{m}} \varphi_{m,\widetilde{m}}(x, x^*)\,. \tag{12}$$

Here $\rho_{kl} = \rho_k - \rho_l$, $\rho_k = \rho^1_k + i\rho^2_k$, and $x = x^1 + ix^2$ is the anharmonic ratio:

$$x = \frac{\rho_{12}\rho_{30}}{\rho_{10}\rho_{32}}\,. \tag{13}$$

The quantum numbers $m$ and $\widetilde{m}$ are the conformal weights of the state $\Psi_{m,\widetilde{m}}(\boldsymbol{\rho}_0)$ belonging to the basic series of the unitary representations of the Moebius group:

$$m = \frac{1}{2} - i\nu + \frac{n}{2}\,,\ \widetilde{m} = \frac{1}{2} - i\nu - \frac{n}{2}\,, \tag{14}$$

where $n$ is the conformal spin, and $d = 1 - 2i\nu$ is the anomalous dimension of the operator $O_{m,\widetilde{m}}(\boldsymbol{\rho}_0)$ describing the compound state [18]. The function $f_{m,\widetilde{m}}(\boldsymbol{\rho}_1,\, \boldsymbol{\rho}_2,\, \boldsymbol{\rho}_3;\, \boldsymbol{\rho}_0)$ is an eigenfunction of the integrals of motion $A$ and $A^*$ , where $A = i^3\rho_{12}\rho_{23}\rho_{31}\partial_1\partial_2\partial_3$ with $\partial_k = \partial/(\partial\rho_k)$ [7]. In the $x$-representation the eigenvalue equation for $A$ takes the form

$$A_{\,m}\varphi_{m,\widetilde{m}}(x, x^*) = \lambda_m\ \varphi_{m,\widetilde{m}}(x, x^*)\ , \tag{15}$$

where $A_m$ can be written in the factorized form [7, 17]

$$A_m = a_{1-m}(x)\, a_m(x)\,,\ a_m(x) = x(1-x)\,(i\partial)^{1+m}\ . \tag{16}$$

Note, that $A_m$ is the ordinary differential operator of the third order

$$A_m = i^3 x(1-x)\Big(x(1-x)\,\partial^3 + (2-m)\,(1-2x)\,\partial^2 - (2-m)(1-m)\partial\Big)\ . \tag{17}$$

We are looking for a solution which is annihilated by the operator $AA^*$. The zero modes of the operator $A_m$ with $\lambda_m = 0$ are 1, $x^m$ and $(1-x)^m$. The corresponding wave function

in the $(x, x^*)$ representation for the state symmetric under the cyclic transmutations $\boldsymbol{\rho}_1 \to \boldsymbol{\rho}_2 \to \boldsymbol{\rho}_3 \to \boldsymbol{\rho}_1$ of the gluon coordinates is

$$\varphi^0_{m,\widetilde{m}}(\boldsymbol{x}) = 1 + (-x)^m(-x^*)^{\widetilde{m}} + (x-1)^m(x^*-1)^{\widetilde{m}}. \tag{18}$$

For even values of the conformal spin this wave function is not normalized and does not correspond to any physical state. However, for odd conformal spins

$$n = m - \widetilde{m} = 2k+1 \tag{19}$$

the above expression vanishes at $x \to 0, 1$ and $\infty$

$$\varphi^0_{m,\widetilde{m}}(x, x^*) \to 0\,, \tag{20}$$

and is normalized [17]. In this case the wave function $f_{m,\widetilde{m}}(\boldsymbol{\rho}_1,\, \boldsymbol{\rho}_2,\, \boldsymbol{\rho}_3;\, \boldsymbol{\rho}_0)$ is anti-symmetric under the pair transmutations $\boldsymbol{\rho}_i \longleftrightarrow \boldsymbol{\rho}_k$ of the gluon coordinates. Therefore, due to requirements of the Bose symmetry of the total wave function, it describes a colourless state with the colour wave function proportional to the structure constant $f_{a_1a_2a_3}$ of the gauge group. This state has positive charge parity and gives a non-vanishing contribution to the structure function $g_2$ for the deep-inelastic scattering of the polarized electron off the polarized proton [17]. The value of the energy turns out to be a half of the energy for the Pomeron with corresponding $m$ and $\widetilde{m}$. The minimal value of it is reached for $m=1$, $\widetilde{m}=0$ or $m=0$, $\widetilde{m}=1$ and equals zero [17].

For the interactions of three (and more) gluons the hamiltonian and the integrals of motion are invariant under duality transformations among the coordinates and momenta of the gluons [7]. Generally this invariance leads to a degeneracy of the spectrum of these operators. Namelly, two eigenfunctions $\varphi^1_{m,\widetilde{m}}(x, x^*)$ and $\varphi^2_{1-m,1-\widetilde{m}}(x, x^*)$ corresponding to the same eigenvalues of $A$ and $A^*$ are related by the duality operator $Q_{m,\widetilde{m}}$ [7]:

$$Q_{m,\widetilde{m}}\varphi^1_{m,\widetilde{m}}(x, x^*) = a_m(x)\, a_{\widetilde{m}}(x^*)\, \varphi^1_{m,\widetilde{m}}(x, x^*) =$$

$$|x(1-x)|^2\,(i\partial)^{1+m}(\,i\partial^*)^{1+\widetilde{m}}\,\varphi^1_{m,\widetilde{m}}(x, x^*) = c\varphi^2_{1-m,1-\widetilde{m}}(x, x^*), \tag{21}$$

where $c$ is an unessential constant.

Starting from the symmetric solution (18), let us use this duality transformation in order to obtain an odderon solution. Since for odd values of the conformal spin $n$ the duality operator changes its sign under the transformation $x \longleftrightarrow 1-x$, the duality transformations lead to relations between totally symmetric and anti-symmetric wave functions $f(\boldsymbol{\rho}_1,\, \boldsymbol{\rho}_2,\, \boldsymbol{\rho}_3;\, \boldsymbol{\rho}_0)$. In the particular case $\lambda = 0$ the anti-symmetric wave function $\varphi^0_{m,\widetilde{m}}(x, x^*)$ is given above in (22), and the symmetric wave function $\varphi^{odd}_{m,\widetilde{m}}(x, x^*)$ describing an odderon state can be obtained from the solution of the equation

$$Q_{m,\widetilde{m}}\,\varphi^{odd}_{m,\widetilde{m}}(x, x^*) = c\varphi^0_{1-m,1-\widetilde{m}}(x, x^*). \tag{22}$$

It is important to take into account that the solution $f^{odd}_{m,\widetilde{m}}(\boldsymbol{\rho}_1,\,\boldsymbol{\rho}_2,\,\boldsymbol{\rho}_3;\,\boldsymbol{\rho}_0)$ includes the propagators of the external gluons. The amputated solution $F^{odd}_{m,\widetilde{m}}(\boldsymbol{\rho}_1,\,\boldsymbol{\rho}_2,\,\boldsymbol{\rho}_3;\,\boldsymbol{\rho}_0)$ with the removed propagators can be written as follows

$$F^{odd}_{m,\widetilde{m}}(\boldsymbol{\rho}_1,\,\boldsymbol{\rho}_2,\,\boldsymbol{\rho}_3;\,\boldsymbol{\rho}_0)=\left|\frac{1}{\rho_{12}\rho_{23}\rho_{31}}\right|^2|A|^2\,f^{odd}_{m,\widetilde{m}}(\boldsymbol{\rho}_1,\,\boldsymbol{\rho}_2,\,\boldsymbol{\rho}_3;\,\boldsymbol{\rho}_0)=$$

$$\left|\frac{1}{\rho_{12}\rho_{23}\rho_{31}}\right|^2\left(\frac{\rho_{23}}{\rho_{20}\rho_{30}}\right)^m\left(\frac{\rho^*_{23}}{\rho^*_{20}\rho^*_{30}}\right)^{\widetilde{m}}\Phi^{odd}_{m,\widetilde{m}}(x,x^*)\,, \tag{23}$$

where $\Phi^{odd}_{m,\widetilde{m}}(x,x^*)$ is obtained to be

$$\Phi^{odd}_{m,\widetilde{m}}(x,x^*)=\left|Q_{m,\widetilde{m}}\right|^2\varphi^{odd}_{m,\widetilde{m}}(x,x^*)=c\,|x(1-x)|^2\,(i\partial)^{2-m}(\;i\partial^*)^{2-\widetilde{m}}\varphi^0_{1-m,1-\widetilde{m}}(x,x^*). \tag{24}$$

With the use of a Fourier transformation one can verify that

$$(i\partial)^{2-m}(\;i\partial^*)^{2-\widetilde{m}}\varphi^0_{1-m,1-\widetilde{m}}(x,x^*)=a\left(\delta^2(x)-\delta^2(1-x)+\frac{x^m x^{*\widetilde{m}}}{|x|^6}\delta^2(\frac{1}{x})\right), \tag{25}$$

where $a$ is a constant. Therefore we obtain

$$F^{odd}_{m,\widetilde{m}}(\boldsymbol{\rho}_1,\,\boldsymbol{\rho}_2,\,\boldsymbol{\rho}_3;\,\boldsymbol{\rho}_0)\sim$$

$$\left|\frac{\rho_{20}\rho_{30}}{\rho^2_{10}\rho^3_{32}}\right|^2\left(\frac{\rho_{23}}{\rho_{20}\rho_{30}}\right)^m\left(\frac{\rho^*_{23}}{\rho^*_{20}\rho^*_{30}}\right)^{\widetilde{m}}\left(\delta^2(x)-\delta^2(1-x)+\frac{x^m x^{*\widetilde{m}}}{|x|^6}\delta^2(\frac{1}{x})\right)\sim$$

$$\frac{E_{m\widetilde{m}}(\rho_{20},\rho_{30})}{|\rho_{23}|^4}\delta^2(\rho_{12})+\frac{E_{m\widetilde{m}}(\rho_{10},\rho_{20})}{|\rho_{12}|^4}\delta^2(\rho_{31})+\frac{E_{m\widetilde{m}}(\rho_{30},\rho_{10})}{|\rho_{31}|^4}\delta^2(\rho_{23}), \tag{26}$$

where

$$E_{m\widetilde{m}}(\boldsymbol{\rho}_{10},\boldsymbol{\rho}_{20})=\left(\frac{\rho_{12}}{\rho_{10}\rho_{20}}\right)^m\left(\frac{\rho^*_{12}}{\rho^*_{10}\rho^*_{20}}\right)^{\widetilde{m}} \tag{27}$$

is the BFKL wave function. Using the fact that $E_{m\widetilde{m}}(\boldsymbol{\rho}_{10},\boldsymbol{\rho}_{20})$ is the eigenfunction of the Kasimir operators $M^2=\rho^2_{12}\partial_1\partial_2$ and $M^{*2}$ of the Moebius group, we can write the odderon solution (26) as follows:

$$F^{odd}_{m,\widetilde{m}}(\boldsymbol{\rho}_1,\,\boldsymbol{\rho}_2,\,\boldsymbol{\rho}_3;\,\boldsymbol{\rho}_0)\sim\sum_{i,k\neq l}\delta^2(\boldsymbol{\rho}_{li})\,|\boldsymbol{\partial}_i|^2\,|\boldsymbol{\partial}_k|^2\,E_{m\widetilde{m}}(\boldsymbol{\rho}_{i0},\boldsymbol{\rho}_{k0})\,, \tag{28}$$

where the summation is performed over all gluon indices $i,\,k,\,l\,=\,1,2,3$ providing that $i,k\neq l$. After the transition to the momentum space we obtain for this vertex function

$$F^{odd}_{m,\widetilde{m}}(\boldsymbol{k}_1,\,\boldsymbol{k}_2,\,\boldsymbol{k}_3)\sim\sum_{i,k\neq l}|\boldsymbol{k}_i+\boldsymbol{k}_l|^2\,|\boldsymbol{k}_k|^2\,E_{m\widetilde{m}}(\boldsymbol{k}_i+\boldsymbol{k}_l,\boldsymbol{k}_k)\,, \tag{29}$$

where

$$E_{m\widetilde{m}}(\boldsymbol{k}_1, \boldsymbol{k}_2) = \int \frac{d^2\rho_1 d^2\rho_2}{(2\pi)^2} \exp\left( i \sum_{r=1}^{2} \boldsymbol{k}_r \boldsymbol{\rho}_r \right) E_{m\widetilde{m}}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) \,. \tag{30}$$

Eq.(29) is the amputated counterpart of (9).

5. Finally let us analyze the coupling of the new odderon state represented by its eigenfunction (9) to the $\gamma^* \to \eta_c$ impact factor (3). Its knowledge will permit us to study scattering processes with odderon exchange, using the odderon Green function. The key point to note is that only the full impact factor $\Phi_{\gamma\to\eta_c}$ in (3) has a "good" infrared behaviour: it vanishes as any $\boldsymbol{k}_i \to 0$. Therefore, inside an integral any individual term $\varphi_-$ will have infrared singularities, but they will cancel if we consider the full sum (3). These singularities are also related to the nature of the conformal invariant eigenfunction of the BFKL pomeron (27): in the momentum representation they contain $\delta$-like pieces [21, 22], corresponding to a constant behaviour in configuration space, as one of the two coordinates $\boldsymbol{\rho}_1$, $\boldsymbol{\rho}_2$ is taken to $\infty$. In a mathematical sense, therefore, the Pomeron eigenfunction is a distribution, and its meaning has to be understood by integrating with some test function. Depending on the space of test functions, one can expect slightly different results. The same must also be true for the action of an operator on (27), e.g. the BFKL Hamiltonian: the result will, again, be a distribution and has to be integrated with a test function. All this is not a merely mathematical observation, but it has a natural physical meaning: the space of test functions in BFKL dynamics is defined by couplings ("impact factors") to colorless objects. In (3) the function $\varphi_-$ alone does not have the normal "good" properties of a colourless object; only the sum of all the terms in (3) defines a "good" function.

Let us take a closer look at the scalar product of $\Phi_{\gamma\to\eta_c}$ with $E_3^{(\nu,n)}$, taking into account the antisymmetry properties of the two building objects, $\varphi_-$ and $E^{(\nu,n)}$. Using the momentum structure in (3) and (9) one finds

$$\begin{aligned}
\langle \Phi_{\gamma\to\eta_c} | E_3^{(\nu,n)} \rangle &= \int d\mu_3 \Phi_{\gamma\to\eta_c}(\{\boldsymbol{k}_i\}) E_3^{(\nu,n)}(\{\boldsymbol{k}_i\}) \\
&= -6 \int d^2\boldsymbol{k} \left[ \varphi_-(\boldsymbol{k}, \boldsymbol{q}-\boldsymbol{k}) - \varphi_-(\boldsymbol{0}, \boldsymbol{q}) \right] \left( \boldsymbol{K}_L \otimes E^{(\nu,n)} \right)(\boldsymbol{k}, \boldsymbol{q}-\boldsymbol{k}) \,,
\end{aligned} \tag{31}$$

where $d\mu_3 = \prod_i d^2\boldsymbol{k}_i \, \delta^{(2)}(\boldsymbol{q} - \sum_i \boldsymbol{k}_i)$, and

$$\begin{aligned}
&\left( \boldsymbol{K}_L \otimes E^{(\nu,n)} \right)(\boldsymbol{k}, \boldsymbol{q}-\boldsymbol{k}) = \\
&\frac{N_c \alpha_s}{2\pi^2} \int d^2\boldsymbol{l} \Big[ \frac{\boldsymbol{l}^2}{\boldsymbol{k}^2 (\boldsymbol{l}-\boldsymbol{k})^2} E^{(\nu,n)}(\boldsymbol{l}, \boldsymbol{q}-\boldsymbol{l}) - \frac{1}{2} \frac{\boldsymbol{k}^2}{\boldsymbol{l}^2 (\boldsymbol{k}-\boldsymbol{l})^2} E^{(\nu,n)}(\boldsymbol{k}, \boldsymbol{q}-\boldsymbol{k}) \Big] \,.
\end{aligned} \tag{32}$$

One sees easily that $\boldsymbol{K}_L$ stands for "half" of the forward BFKL kernel; adding the corresponding expression for $\boldsymbol{K}_R$ one obtains the BFKL kernel, but without the local piece. Due to the antisymmetry of $E^{(\nu,n)}$, the local term in the BFKL kernel is giving zero contribution. Therefore, in the first term of the integrand in (31), using the antisymmetry of $\varphi_-$, one could also include one half of this local BFKL-piece, such that $\boldsymbol{K}_L$ really represents "half"

of the full BFKL kernel. Ignoring all potential divergences, one might naively expect that (32) should, basically, lead to $\chi E^{(\nu,n)}$, and our scalar product $\Phi \otimes E^{(\nu,n)}$ equals $\chi\varphi_- \otimes E^{(\nu,n)}$. This expectation turns out to be correct, but the argument is rather subtle. First, one notices that the first and the second integrand in (32) by themselves lead to infrared divergent integrals, whereas the scalar product of $\varphi_-$ with $E^{(\nu,n)}$ is convergent. So it is clear that in the integration we are not allowed simply to use $\boldsymbol{K}_{BFKL} \otimes E^{(\nu,n)} = \chi_{\nu,n} E^{(\nu,n)}$. The divergent pieces and, possibly also finite parts, would be lost. All these complications would not be visible if $\varphi_-$ would be a "good" function.

In order to see the resolution to this puzzle, we calculate $\boldsymbol{K}_L \otimes E^{(\nu,n)}$ explicitly. Rather than presenting details of this calculation we only quote the main result. In the coordinate representation we find:

$$(\boldsymbol{K}_L \otimes E^{(\nu,n)})(\rho_1, \rho_2) = \frac{1}{2}\chi_{\nu,n} E^{(\nu,n)}(\rho_1, \rho_2) + C \lim_{\rho_1 \to \infty} E^{(\nu,n)}(\rho_1, \rho_2)\,, \tag{33}$$

where $C$ contains some infinite (after removing the infrared regularization) contributions and also finite pieces. In momentum space, this relation corresponds to the presence of some extra $\delta$ function-like pieces. It turns out that, as it should be, these extra terms give a contribution which is exactly cancelled by the second integrand in (31) (the subtraction term). It is only after these cancellations that finally we can write

$$\langle \Phi_{\gamma\to\eta_c} | E_3^{(\nu,n)} \rangle = \sqrt{-3\chi_{\nu,n}} \int d^2\boldsymbol{k}\, \varphi_-(\boldsymbol{k}, \boldsymbol{q}-\boldsymbol{k}) E^{(\nu,n)}(\boldsymbol{k}, \boldsymbol{q}-\boldsymbol{k})\,. \tag{34}$$

Thus the matrix element of our odderon solution is similar to the corresponding matrix element of the pomeron solution: this is related to our interpretation of the new odderon as a compound state of "f" and "d" reggeized gluons. Note that the degeneracy between the "f" and the "d" gluons is exact in the large $N_c$ limit, and the duality [7] can be considered as a manifestation of this symmetry. Finally we just remark that in calculating the norm of $E_3^{(\nu,n)}$ the $\delta$-like pieces do not play any role. In fact, in place of $\Phi_{\gamma\to\eta_c}$ one has the amputated odderon function given in (29) and, therefore, in place of $\varphi_-$ the amputated pomeron eigenfunction which has "good" properties.

6. We have presented a new set of eigenfunctions of the odderon equation which is characterized by a spectrum with a maximum intercept at 1. It is remarkable that symmetry structure of this solution has been suggested by the impact factor $\Phi_{\gamma\to\eta_c}$, to which the odderon couples, and by the Pomeron→ two odderon vertex $W$ which came our from the study of the six gluon amplitude. At the same time one can use the duality symmetry of the 3 gluon Hamiltonian to rederive this solution. This derivation shows the interconnection with solutions of different symmetry properties. Finally we have shown how to calculate the scalar product of the eigenfunction with the impact factor. This opens the possibility to calculate numerically the contribution of the new odderon states to some of those processes which have already been studied to probe the QCD odderon. Work in this direction is in progess.

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